

# Knowledge-based modelling for dynamic electrical tomography in medicine and process engineering

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## Abstract

In all applications of tomography, a section through an object is imaged using measurements taken from outside or on the boundary of the object. There are many applications of this technique in areas such as medical imaging, industrial process monitoring and geophysics. Although many application only involve single snap-shots, more challenging examples involve sequences of multi-component data and allow the investigation of dynamically evolving processes.

This paper will describe two examples: the mixing of liquids in a tank and the movement of heart and lungs in the human thorax. Different approaches to modelling, including choice of parameterisation and spatial and temporal prior modelling will be discussed. Results, obtained using MCMC techniques will be presented. In particular, the flexibility of the modelling approach, allowing a variety of prior knowledge to be incorporated and a variety of output summaries, will be highlighted.

## Introduction

In tomography the aim is to reconstruct the inside of an object using measurements taken on the outside of the object. There are many applications of this approach in areas such as medical imaging, industrial process monitoring and geophysics.

Here the focus is on electrical impedance tomography (EIT) where electrodes are attached to the boundary of the object and, whilst currents are applied between some electrodes, voltage measurements are recorded between other electrodes. The relationship between parameters (resistivities) and data (voltages) is nonlinear, and in a classical setting estimation requires regularization to overcome numerical instability. Calculation of voltages from a specified resistivity distribution requires the numerical solution of a PDE (see Vauhkonen *et al.* 2001). The Bayesian approach provides a convenient setting in which to define, control and interpret regularization and has wide ranging applicability in many imaging problems.

Although a pixel-based formulation is widely used and provides a generic approach, it is not always the most useful. In particular, problem-specific formulations, using information regarding known structure or physiological behaviour introduce reliable regularization through parameter reduction and allow important process quantities to be directly investigated without the need for post-processing of image reconstructions. For general background to Bayesian modelling and MCMC estimation see Besag *et al.* (1995) and Liu (2001). The various spatial and temporal prior models are combined with the likelihood distribution by Bayes' theorem and parameter estimation is then based on the posterior distribution using MCMC methods.

**Acknowledgement:** The authors thank the EPSRC for financial support (GR/R22148/01).

## Example 1: Mixing tank

An experiment was undertaken to investigate the mixing of two liquids in a laboratory tank (see West *et al* 2004b). Initially the tank contains only tap water which is stirred. While the water is still revolving an aliquot of salty water is injected into the flow. The aliquot will gradually disperse and mix eventually leaving homogeneous, but slightly salty, tank contents.

At different stages through the experiment our prior knowledge regarding the spatial and temporal smoothness changes. During the first few frames the tank contains a homogeneous solution (tap water) and the resistivity should be spatially and temporally very smooth. These frames have been used for calibration to estimate the contact impedances and the background resistivity. In the second stage an aliquot of low resistivity saline solution (tap water and potassium chloride) is injected into the tank and temporal smoothing breaks down completely. In the spatial distribution we expect a resistivity discontinuity at the aliquot boundary, with high smoothness elsewhere. As the aliquot begins to disperse there is moderate temporal smoothness and increasing spatial smoothness. At the end of this stage the solution should again be well mixed and so the endpoint will also be a homogeneous solution with resistivity between that of the initial contents and the saline solution.

## Example 2: Thorax

A time sequence of resistivity distributions is considered which covers the whole heart–lung cycle. See West *et al.* (2003b) and reference therein for further details. Previous work has required post-processing of reconstructed images and the use of ad-hoc regions of interest to identify organ outline and hence estimate clinical parameters.

Here explicit geometric models have been used which also incorporate temporal smoothness. The model states that the resistivity distribution is composed of a constant background and fixed spine, but with contracting and expanding regions for the heart and lungs. This provides very specific prior knowledge about the resistivity distribution – not just that it has discontinuities. Further, it is assumed that the location of the heart is known, but that its size and resistivity are to be estimated. Each of the lung is assumed to have a minimum size, corresponding to maximal expiration, but the actual shape, sizes and resistivities are unknown and will be estimated. As the process evolves the state is expected to change slowly and smoothly. This is modelled using temporal priors which are propagated within the frame sequence. Output includes clinically relevant quantities such as lung residual capacity, and heart ejection fractions.

## References

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